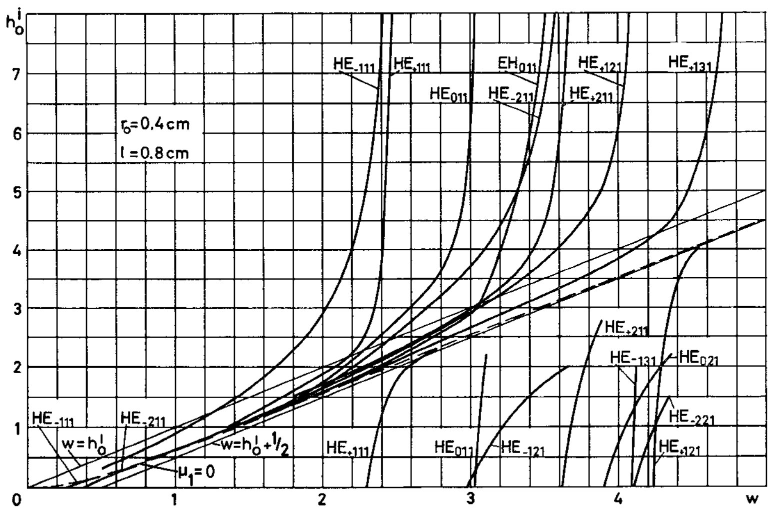

Fig. 2. Calculated h_0^i/w tuning curves of some modes.

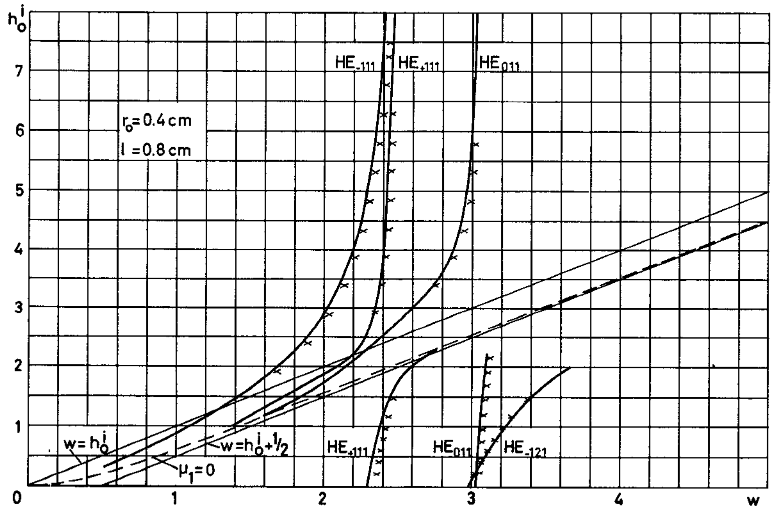

Fig. 3. Experimental results and the corresponding calculated h_0^i/w tuning curves of a few modes.

bandstop filters, and selective modulators with directional characteristic. The magnetodynamic modes are also useful for material measurements especially in the mm-wave region because of the relatively large dimensions.

Acknowledgment

The numerical calculations were performed at the Rechenzentrum of the Technische Hochschule, Aachen. The authors wish to thank D. Haupt and his staff and are indebted to Prof. H. Döring for encouragement and advice.

H.-D. Godtmann
W. Haas
5032 Efferen/b. Koln
Dreikoenigenstr 2, Germany

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Relative Humidity Effects in Microwave Resonators

Several factors affect the frequency stability of microwave resonators. One important factor is the stability of the dielectric medium which fills the resonator. It is the purpose of this correspondence to analyze the effect of the change in dielectric constant upon the resonant frequency of a sealed, air-filled resonator.

Earlier papers have dealt with the effect of the dielectric constant on the propagation of microwaves and its application to the shift of resonant frequency in an open resonator.[1,2] Most of the work is based on an empirical equation which relates the dielectric constant of air to temperature and partial pressures. Since the relationship of the partial pressures is significantly different in open and in closed systems, it is important to analyze the frequency shift in resonators with this in mind.

Montgomery's nomograph shown in Fig. 4, which is the one most frequently referred to on this topic, is most useful for an open resonator operating at high relative humidities and elevated temperatures. Often, however, a resonator is sealed at moderate humidities and room temperature, and completely erroneous results can occur if one tries to extrapolate the results of the nomograph to a closed system. This is because, as the temperature is increased in an open system, the partial pressure of the air increases significantly over the partial pressure of the water vapor, giving a large net increase in dielectric constant and hence a large decrease in resonant frequency.

In a closed system, however, the partial pressure of the air and water vapor increases at a smaller linear rate which is cancelled by the increase in temperature. The net result is that there is a small decrease in dielectric constant which slightly raises the resonant frequency.

A similar situation exists at lower temperatures but is compounded by the fact that, in the closed system, 100 percent relative humidity is soon reached and precipitation of the water vapor occurs with attendent large frequency shift.

In order to analyze the situation, let us consider the equation

$$f\lambda = (\mu\epsilon)^{-1/2}. \tag{1}$$

With the assumptions of Montgomery[1] (i.e., medium does not introduce excessive or variable loss), one can derive

$$\frac{\Delta f}{f} = -\frac{1}{2}\frac{\Delta k_e}{k_e} \tag{2}$$

Manuscript received November 1, 1966; revised January 27, 1967, and March 20, 1967.

[1] C. G. Montgomery, *Technique of Microwave Measurements*. New York: McGraw-Hill, 1947, pp. 297, 390, 391.

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where k_e is the relative dielectric constant which we would like in terms of temperature in order to get $\Delta f/f$ in useful terms. Montgomery presents the empirical equation (3) which relates the dielectric constant of air to temperature and partial pressures of dry air and water vapor.[1]

$$k_e = 1 + 1.57 \times 10^{-6} \frac{P_a}{T} + 1.35 \times 10^{-6} \cdot \left(1 + \frac{5580}{T}\right) \frac{P_w}{T} \quad (3)$$

where

P_a = partial pressure of dry air in Newtons per square meter
P_w = partial pressure of water vapor in Newtons per square meter
T = absolute temperature = 273.16°K + T°C.

However, $P_x = m_x/M_x\ RT/V$ for a mass of m_x grams of a gas of molecular mass M_x in a container of volume V, m^3 at a temperature T°K. Substituting the values for 1 mole at standard temperature and pressure for

$P = 10.15 \times 10^4$ N/m²
$V = 22.421 \times 10^{-3}$ m³ — volume
$T = 273.16$°K — temperature
$M = M_x$ g — mass,

the universal gas constant, R, is

$$R = 6.25 \times 10^{-5} \frac{\text{m}^4}{\text{K}^\circ\ \text{g}} \cdot$$

Then (3) can be rewritten as

$$k_e = 1 + k_a m_a + k_w m_w \left(1 + \frac{5580^\circ\ \text{K}}{T}\right) \quad (4)$$

where

$$k_a = 4.53 \times 10^{-7}\ \text{m}^3/\text{g}$$
$$k_w = 6.25 \times 10^{-7}\ \text{m}^3/\text{g}$$

m_a, m_w = specific mass of air and water, respectively, in g/m³.

It is clear that the mass of air remains constant in a closed system. It is also apparent that the mass of water vapor, m_w, in a closed system will remain constant until, at lower temperatures, 100 percent relative humidity is reached. It is at 100 percent relative humidity (dew point) at which precipitation occurs and the dielectric constant (and hence frequency) begins to change drastically. Thus it is a simple matter to calculate Δk_e from (4). Let

$$k_e = 1 + k_a m_a + k_w m_w \left(1 + \frac{5580^\circ\ \text{K}}{T_0}\right)$$

and

$$k_e' = 1 + k_a m_a' + k_w m_w' \left(1 + \frac{5580^\circ\ \text{K}}{T_0'}\right).$$

Then

$$\Delta k_e = k_w \left[m_w \left(1 + \frac{5580^\circ\ \text{K}}{T_0}\right) - m_w' \left(1 + \frac{5580^\circ\ \text{K}}{T_0'}\right)\right]. \quad (5)$$

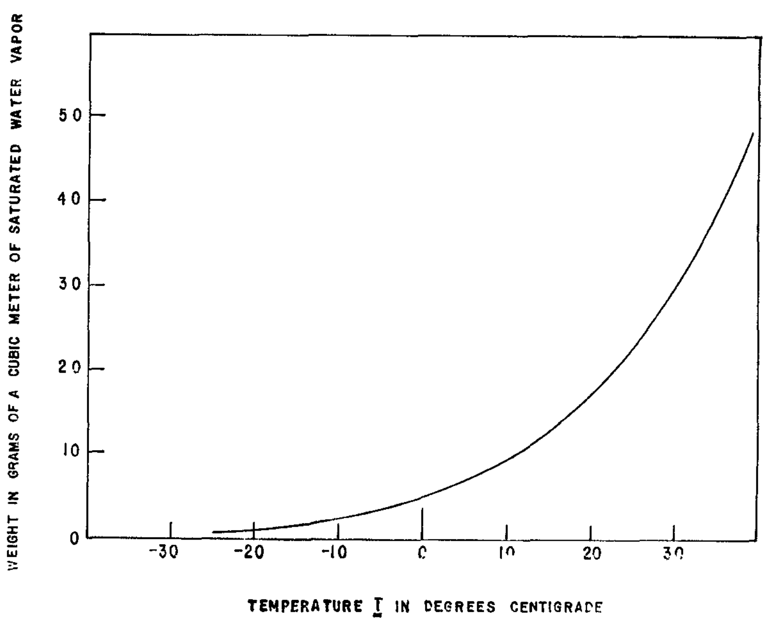

Fig. 1. The weight in grams per cubic meter of saturated water vapor as a function of temperature.

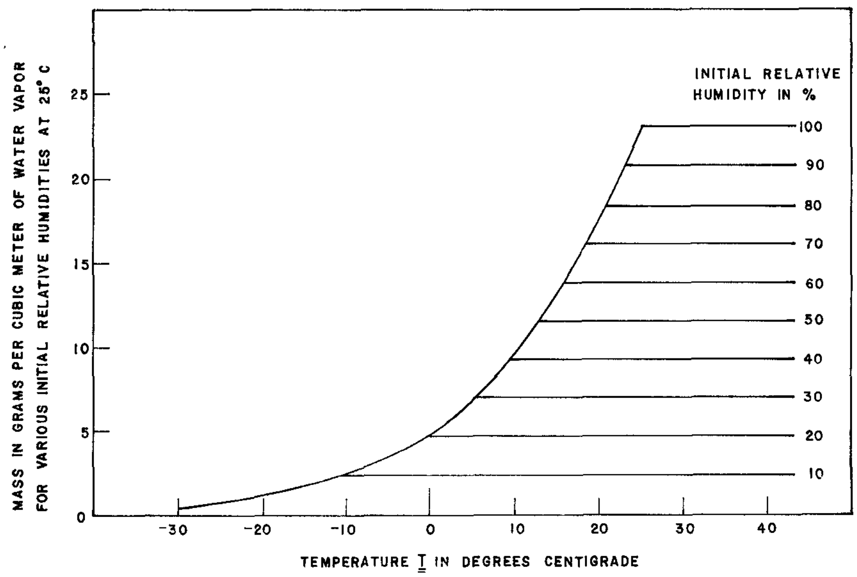

Fig. 2. Mass in grams per cubic meter of water vapor in a cavity sealed at 25°C for several initial values of relative humidity.

As an example of the utility of (5), consider a resonator sealed at a T_0 of 25°C. The specific mass of air in the sealed cavity will vary from 1.18×10^3 at 0 percent relative humidity to 1.15×10^3 g/m³ at 100 percent relative humidity, but will remain constant after sealing. Assume that the relative humidity at the time of sealing was 60 percent. The concept of dealing with masses of water is convenient, since relative humidity is defined as the ratio of the actual amount of water vapor in the air to the amount of water vapor the air could hold at 100 percent relative humidity (dew point) at the temperature of interest. From Fig. 1 we see that there are 23.0 g/m³ of water in a saturated mixture of air and water at 25°C. Hence, for 60 percent relative humidity, there would be 13.824 g/m³ of water vapor. For any temperature other than 25°C, we can now calculate $\Delta f/f$ from (2) and (5) and find it to be $-0.0241\ (/T_0 - 1/T_0')$ as long as m_w is constant. It can be seen from Fig. 2 that m_w remains constant until 100 percent relative humidity is reached, at which point precipitation occurs and m_w decreases. Figure 3 shows a family of curves for several different initial relative humidities, including the assumed case of 60 percent, in a resonator sealed at 25°C. If the cavity is sealed at a different initial temperature or if it is not a

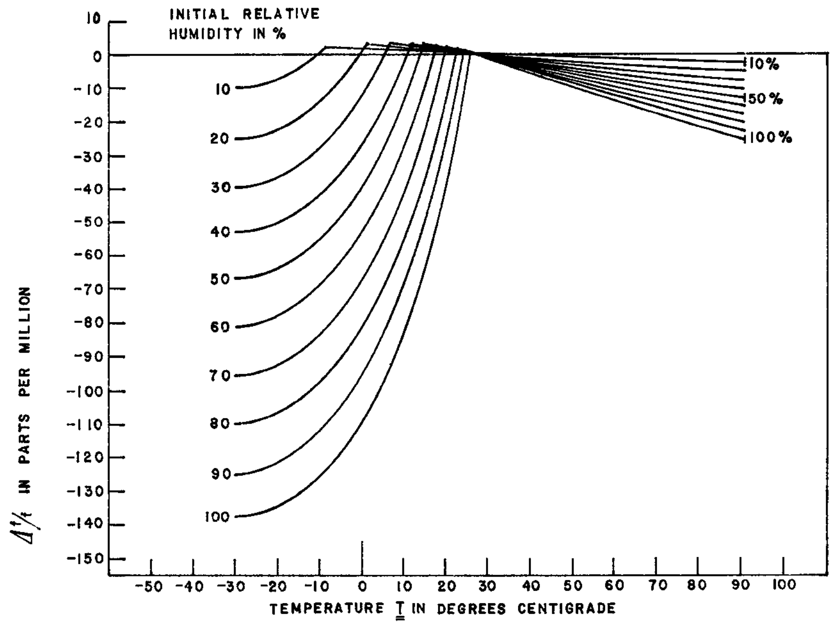

Fig. 3. Frequency shift as a function of temperature for a cavity sealed at 25°C for several different initial relative humidities. Curves apply to a closed system only. Values are to be added to measured values.

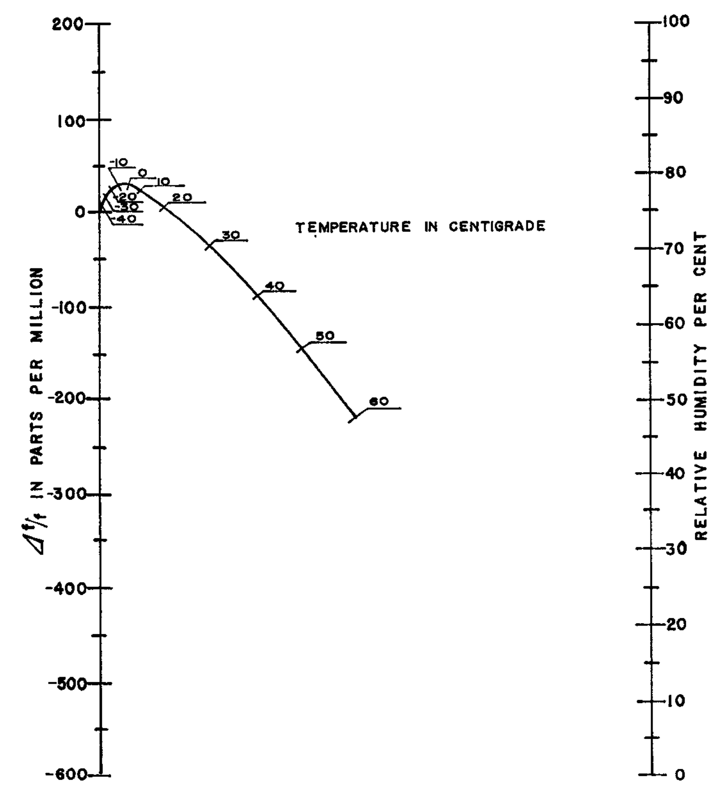

Fig. 4. Nomograph for computing frequency shift as a function of temperature and humidity for an open cavity. $\Delta f/f$ values are read from the intersection of the $\Delta f/f$ scale and the line which joins temperature with relative humidity. Nomograph values are to be added to measured values. (After Montgomery.)

closed system, then all calculations in (2) and (4) must be carried out for these conditions.

Let us now compare the results of Montgomery's nomograph, shown in Fig. 4, to those obtained for the closed system. For both cases let the initial condition of the cavity be 60 percent relative humidity at 25°C and 1 atmosphere. Assume that the frequency shift is desired for the case in which the final temperature is -5°C and the relative humidity is 100 percent. Considering the closed system first, it is obvious that the initial conditions are satisfied by the 60 percent line of Fig. 3. Since the sealed cavity forms a closed system, the same line will satisfy the final conditions of 100 percent relative humidity at -5°C.

From Fig. 3 we see that the frequency shift is plus 62 PPM (since values on the graph are to be added to measured values). Entering the nomograph of Fig. 4 we see that the predicted frequency shift is minus 30 PPM. The reason for the difference in values is due to the fact that the dielectric constant changes minus 90 PPM due to the change of the mass of dry air, and plus 60 PPM for the water vapor in the open system, resulting in a net shift of minus 30 PPM.

It should be pointed out that in actual laboratory measurements there are many other errors to be accounted for. For example, it is well known that water vapor causes an increased absorption level in the K band and that these losses will cause a frequency shift. In addition, at lower temperatures, water precipitation on resonator walls will cause a frequency shift which must be handled with perturbation techniques. The thermal expansion of the cavity walls must also be accounted for.

Actually, for resonators built from conventional materials (aluminum, brass, etc.), the error due to thermal expansion of the walls far exceeds the error introduced by humidity effects. Recently developed resonators built from low-expansion glass ceramics, however, exhibit thermal stabilities of several parts in 10^8. In such resonators, the humidity effect can far overshadow the thermal expansion of the walls.

Still another problem which must be dealt with is the differential of pressure from inside to outside of the resonator which causes mechanical deformation.[3] All of these factors as well as several other minor ones have made it very difficult to precisely confirm the above analysis experimentally. Nevertheless, sufficient work has been performed to indicate that the above treatment is correct.

D. M. Smith
Corning Glass Works
Raleigh, N. C.

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Optimum Elliptic-Function Filters for Distributed Constant Systems

***Abstract*—It is shown from an example that the derivation of distributed constant filters from lumped-element filters through Kuroda's transformation leads to less selectivity than can be obtained with the same number of elements, when the unit elements are also made to contribute to the filtering.**

Manuscript received January 5, 1967; revised March 27, 1967.